Single module identification – partial measurements

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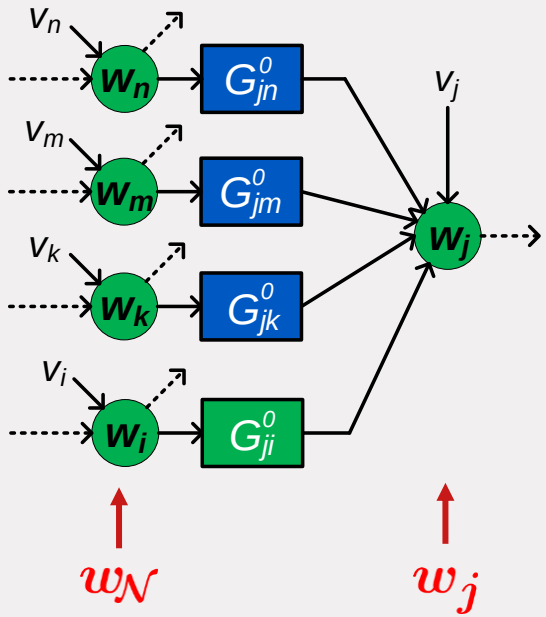
Single module identification

Full MISO situation:

Measure output w_j of target module and all in-neighbors $w_{\mathcal{N}}$ of w_j



Multi-input single-output identification problem

addressed by either a direct or indirect method

Question:

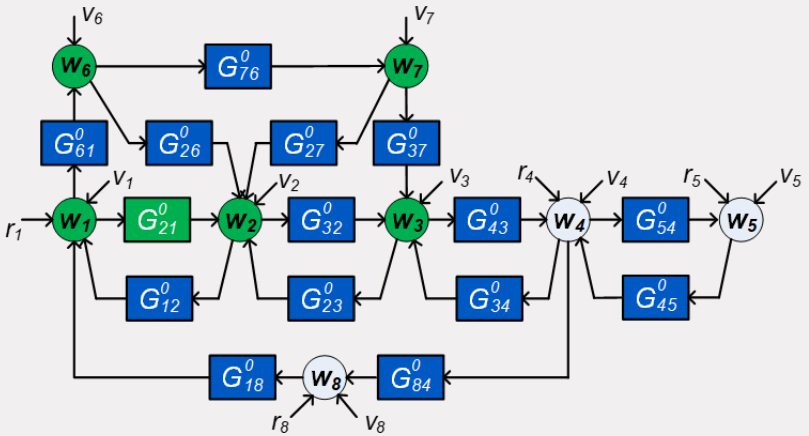
Can we reduce the number of input signals?

Do all in-neighbors of w_j need to be included?

Single module identification

4 input nodes to be measured:

Can we do with less?



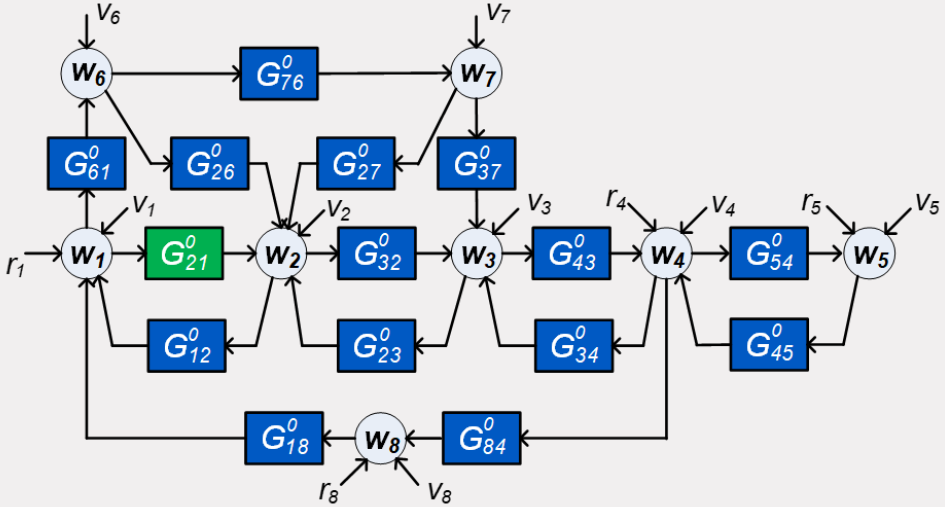
Network immersion [1]

- An **immersed network** is constructed by removing node signals, but leaving the remaining node signals invariant
- Modules and disturbance signals are adapted
- Abstraction through variable elimination (Kron reduction[2] in network theory).

[1] A. Dankers. PhD Thesis, 2014.

[2] F. Dörfler and F. Bullo, IEEE Trans. Circuits and Systems I (2013)

Immersion


v_6
v_7
w_6
G^0_{76}
w_7
G^0_{61}
G^0_{26}
G^0_{27}
G^0_{37}
v_1
v_2
v_3
r_4
v_4
r_5
v_5
r_1
w_1
G^0_{21}
w_2
G^0_{32}
w_3
G^0_{43}
w_4
G^0_{54}
w_5
G^0_{12}
G^0_{23}
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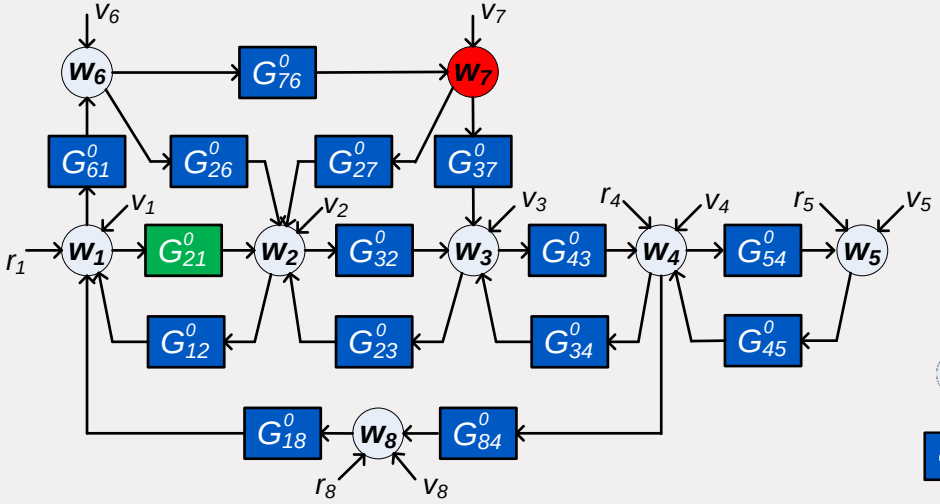
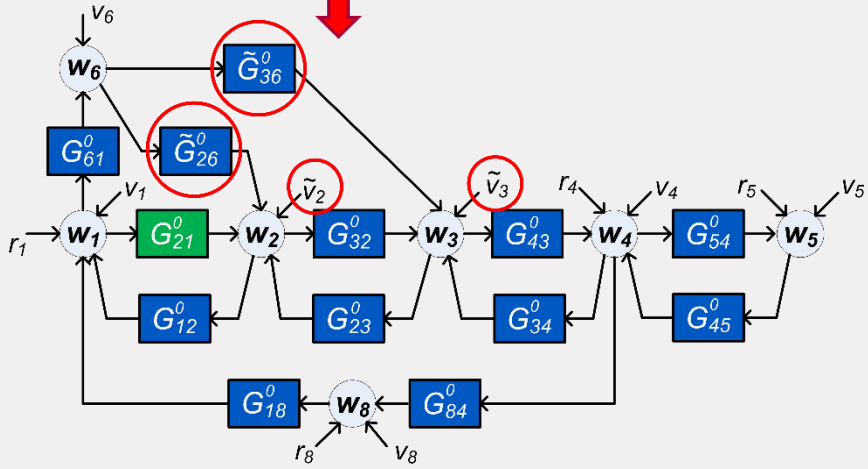
G^0_{45}
G^0_{18}
w_8
G^0_{84}
r_8
v_8

Immersion



Immersing w_7



Algebraic formulation of immersion

$$\begin{bmatrix} w_j \\ w_{\mathcal{D}} \\ w_{\mathcal{Z}} \end{bmatrix} = \begin{bmatrix} 0 & G_{j\mathcal{D}} & G_{j\mathcal{Z}} \\ G_{\mathcal{D}j} & G_{\mathcal{D}\mathcal{D}} & G_{\mathcal{D}\mathcal{Z}} \\ G_{\mathcal{Z}j} & G_{\mathcal{Z}\mathcal{D}} & G_{\mathcal{Z}\mathcal{Z}} \end{bmatrix} \begin{bmatrix} w_j \\ w_{\mathcal{D}} \\ w_{\mathcal{Z}} \end{bmatrix} + \begin{bmatrix} v_j \\ v_{\mathcal{D}} \\ v_{\mathcal{Z}} \end{bmatrix} + \begin{bmatrix} u_j \\ u_{\mathcal{D}} \\ u_{\mathcal{Z}} \end{bmatrix}$$

Immersion of nodes $w_{\mathcal{Z}} \Longrightarrow$ remove nodes $w_{\mathcal{Z}}$ from the equations through Gaussian elimination:

$$(I - G_{\mathcal{Z}\mathcal{Z}})w_{\mathcal{Z}} = G_{\mathcal{Z}j}w_j + G_{\mathcal{Z}\mathcal{D}}w_{\mathcal{D}} + v_{\mathcal{Z}} + u_{\mathcal{Z}}$$

Substituting $w_{\mathcal{Z}}$ in the above equation delivers:

$$\begin{bmatrix} w_j \\ w_{\mathcal{D}} \end{bmatrix} = \begin{bmatrix} 0 & \breve{G}_{j\mathcal{D}} \\ \breve{G}_{\mathcal{D}j} & \breve{G}_{\mathcal{D}\mathcal{D}} \end{bmatrix} \begin{bmatrix} w_j \\ w_{\mathcal{D}} \end{bmatrix} + \begin{bmatrix} \breve{v}_j \\ \breve{v}_{\mathcal{D}} \end{bmatrix} + \begin{bmatrix} \breve{u}_j \\ \breve{u}_{\mathcal{D}} \end{bmatrix}$$

while $w_j, w_{\mathcal{D}}$ remain invariant.

Algebraic formulation of immersion

$$\begin{bmatrix} w_j \\ w_\mathcal{D} \\ w_\mathcal{Z} \end{bmatrix} = \begin{bmatrix} 0 & G_{j\mathcal{D}} & G_{j\mathcal{Z}} \\ G_{\mathcal{D}j} & G_{\mathcal{D}\mathcal{D}} & G_{\mathcal{D}\mathcal{Z}} \\ G_{\mathcal{Z}j} & G_{\mathcal{Z}\mathcal{D}} & G_{\mathcal{Z}\mathcal{Z}} \end{bmatrix} \begin{bmatrix} w_j \\ w_\mathcal{D} \\ w_\mathcal{Z} \end{bmatrix} + \begin{bmatrix} v_j \\ v_\mathcal{D} \\ v_\mathcal{Z} \end{bmatrix} + \begin{bmatrix} u_j \\ u_\mathcal{D} \\ u_\mathcal{Z} \end{bmatrix}$$

Immersion of nodes $w_\mathcal{Z} \Longrightarrow$ remove nodes $w_\mathcal{Z}$ from the equations through Gaussian elimination:

$$(I - G_{\mathcal{Z}\mathcal{Z}})w_\mathcal{Z} = G_{\mathcal{Z}j}w_j + G_{\mathcal{Z}\mathcal{D}}w_\mathcal{D} + v_\mathcal{Z} + u_\mathcal{Z}$$

$$\begin{aligned} w_j &= G_{j\mathcal{D}}w_\mathcal{D} + G_{j\mathcal{Z}}(I - G_{\mathcal{Z}\mathcal{Z}})^{-1}[G_{\mathcal{Z}j}w_j + G_{\mathcal{Z}\mathcal{D}}w_\mathcal{D} + v_\mathcal{Z} + u_\mathcal{Z}] + v_j + u_j \\ w_\mathcal{D} &= G_{\mathcal{D}j}w_j + G_{\mathcal{D}\mathcal{D}}w_\mathcal{D} + G_{\mathcal{D}\mathcal{Z}}(I - G_{\mathcal{Z}\mathcal{Z}})^{-1}[G_{\mathcal{Z}j}w_j + G_{\mathcal{Z}\mathcal{D}}w_\mathcal{D} + v_\mathcal{Z} + u_\mathcal{Z}] + v_\mathcal{D} + v_\mathcal{Z} \end{aligned}$$

Algebraic formulation of immersion

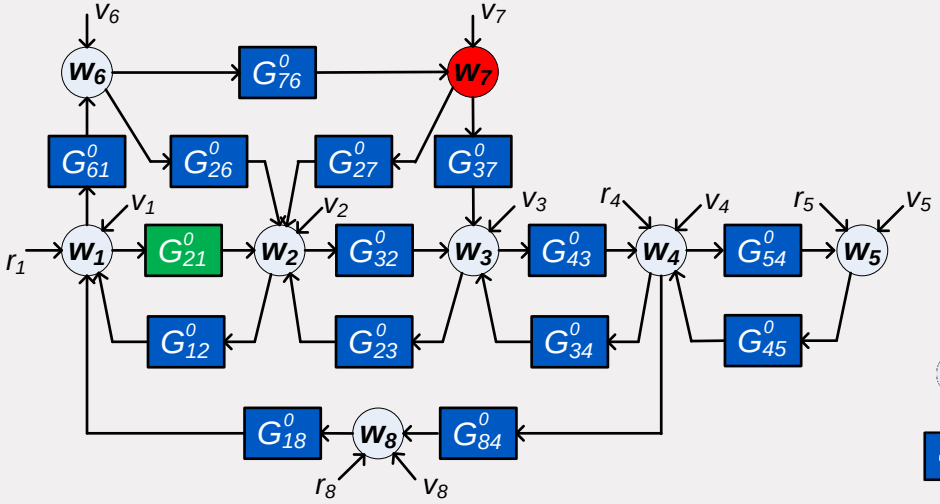
$$
\begin{aligned}
w_j &= G_{j\mathcal{D}} w_{\mathcal{D}} + G_{j\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} [G_{\mathcal{Z}j} w_j + G_{\mathcal{Z}\mathcal{D}} w_{\mathcal{D}} + v_{\mathcal{Z}} + u_{\mathcal{Z}}] + v_j + u_j \\
w_{\mathcal{D}} &= G_{\mathcal{D}j} w_j + G_{\mathcal{D}\mathcal{D}} w_{\mathcal{D}} + G_{\mathcal{D}\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} [G_{\mathcal{Z}j} w_j + G_{\mathcal{Z}\mathcal{D}} w_{\mathcal{D}} + v_{\mathcal{Z}} + u_{\mathcal{Z}}] + v_{\mathcal{D}} + v_{\mathcal{Z}}
\end{aligned}
$$

$$
\begin{bmatrix} w_j \\ w_{\mathcal{D}} \end{bmatrix} = \begin{bmatrix} 0 & \breve{G}_{j\mathcal{D}} \\ \breve{G}_{\mathcal{D}j} & \breve{G}_{\mathcal{D}\mathcal{D}} \end{bmatrix} \begin{bmatrix} w_j \\ w_{\mathcal{D}} \end{bmatrix} + \begin{bmatrix} \breve{v}_j \\ \breve{v}_{\mathcal{D}} \end{bmatrix} + \begin{bmatrix} \breve{u}_j \\ \breve{u}_{\mathcal{D}} \end{bmatrix}
$$

$$
\begin{aligned}
\breve{G}_{j\mathcal{D}} &= [I - G_{j\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} G_{\mathcal{Z}j}]^{-1} [G_{j\mathcal{D}} + G_{j\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} G_{\mathcal{Z}\mathcal{D}}] \\
\breve{G}_{\mathcal{D}j} &= G_{\mathcal{D}j} + G_{\mathcal{D}\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} G_{\mathcal{Z}j} \\
\breve{G}_{\mathcal{D}\mathcal{D}} &= G_{\mathcal{D}\mathcal{D}} + G_{\mathcal{D}\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} G_{\mathcal{Z}\mathcal{D}} \\
\breve{v}_j &= v_j + [I - G_{j\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} G_{\mathcal{Z}j}]^{-1} G_{j\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} v_{\mathcal{Z}} \\
\breve{v}_{\mathcal{D}} &= v_{\mathcal{D}} + G_{\mathcal{D}\mathcal{Z}} (I - G_{\mathcal{Z}\mathcal{Z}})^{-1} v_{\mathcal{Z}}
\end{aligned}
$$

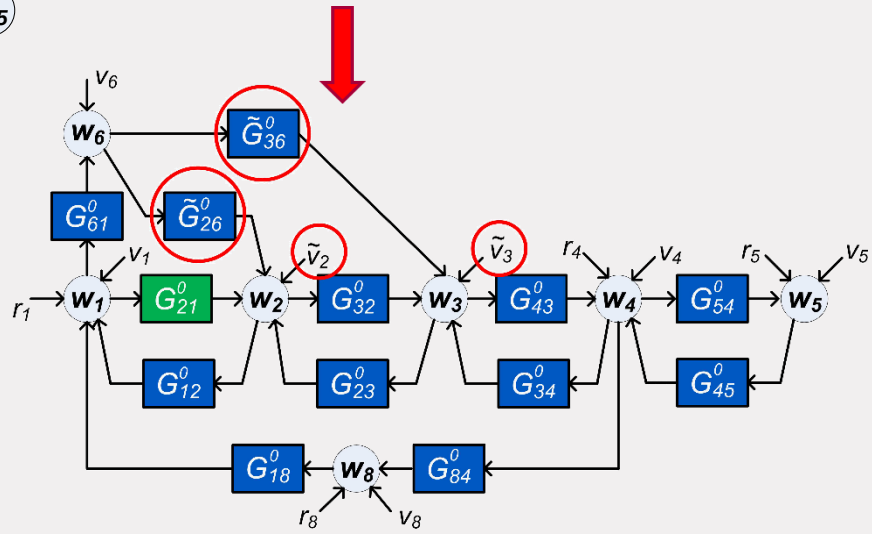
TU/e

Immersion



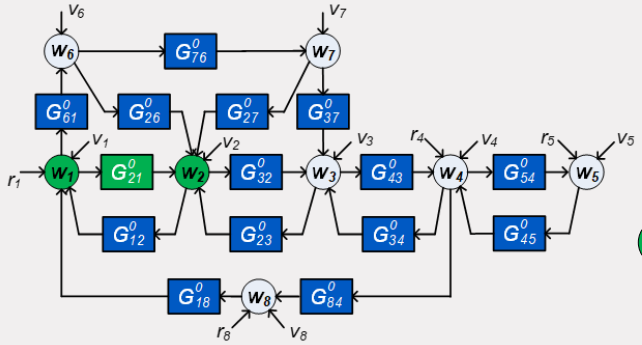
When does immersion of a node leave G^0_{21} invariant?

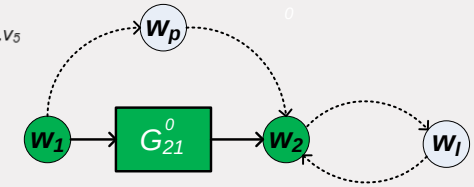
Immersing w_7



Immersion

When does immersion of a node leave G_{21}^0 invariant?





Proposition

Consider an immersed network where w_1 and w_2 are retained.

Then $\check{G}_{21}^0 = G_{21}^0$ if

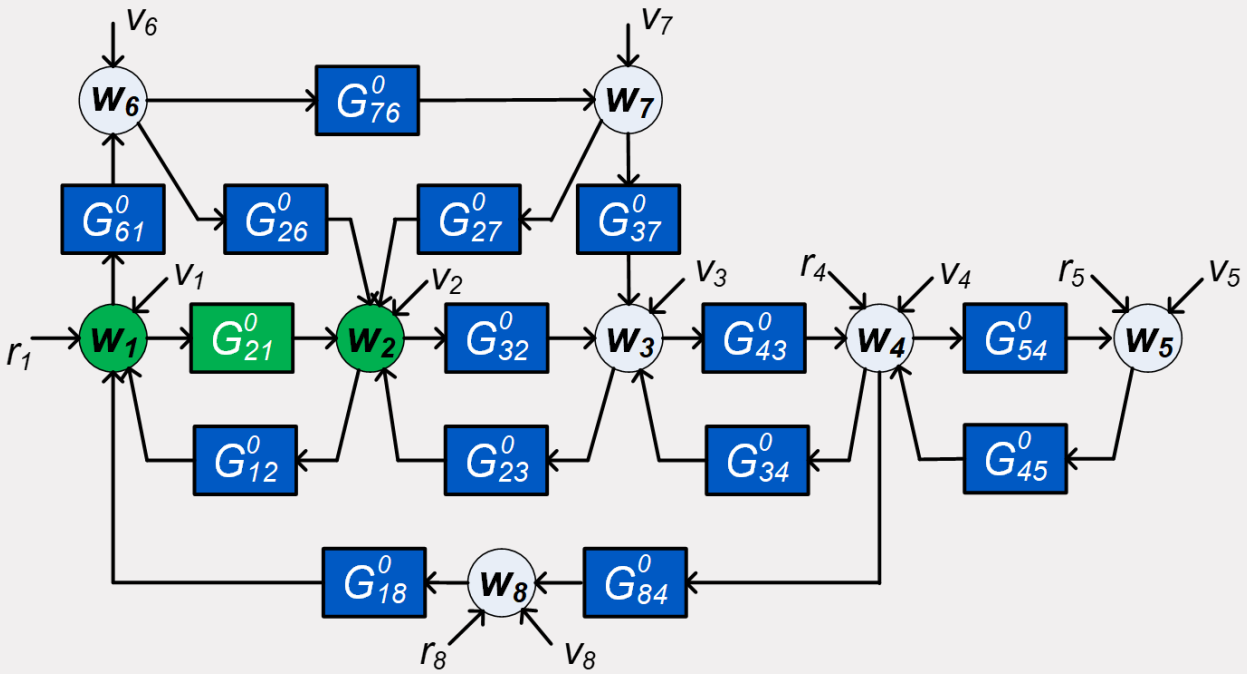
a) Every path $w_1 \to w_2$ other than the one through G_{21}^0 passes through a node that is retained. (parallel paths)

b) Every path $w_2 \to w_2$ passes through a node that is retained. (loops around the output)

A.G. Dankers et al., *IEEE Trans. Automatic Control, 61*, 937-952, 2016.

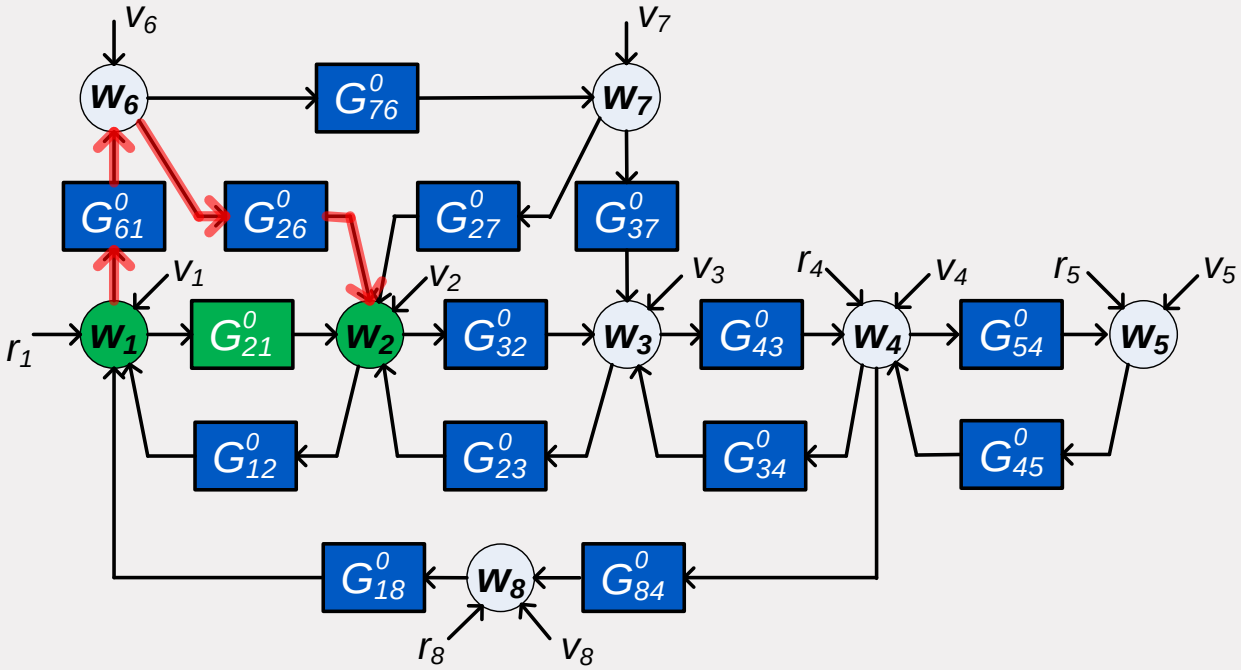
Single module identification

parallel paths, and loops around the output



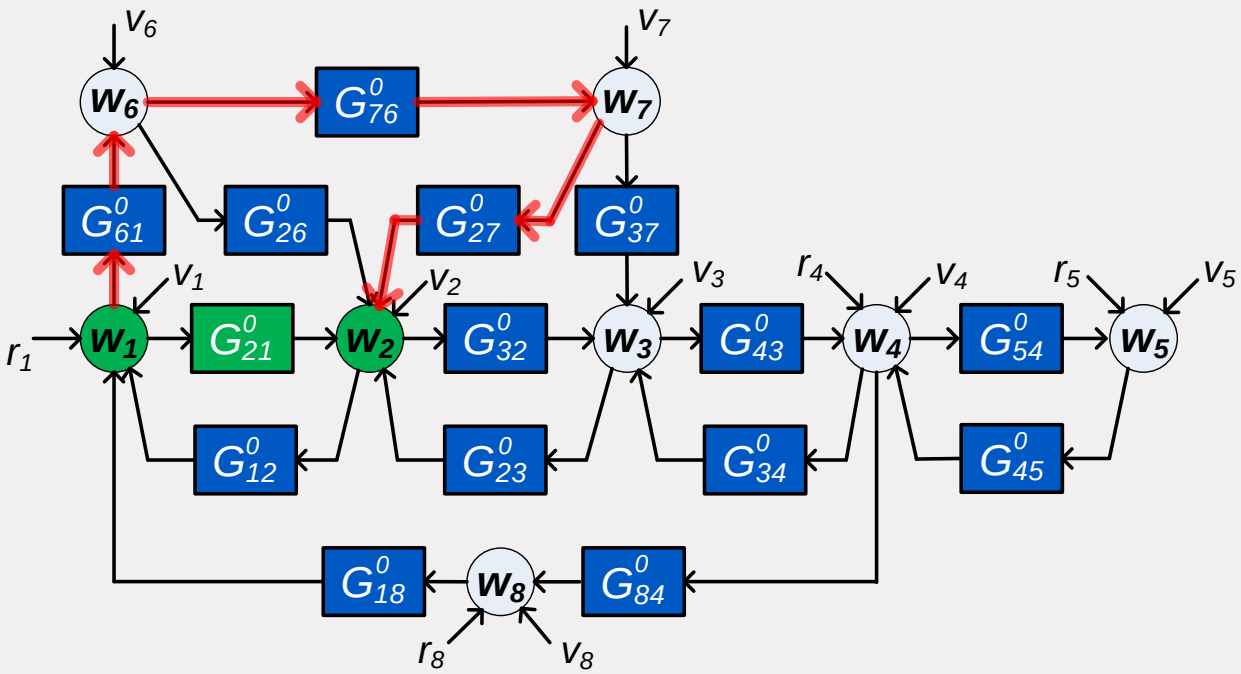
Single module identification

parallel paths, and loops around the output



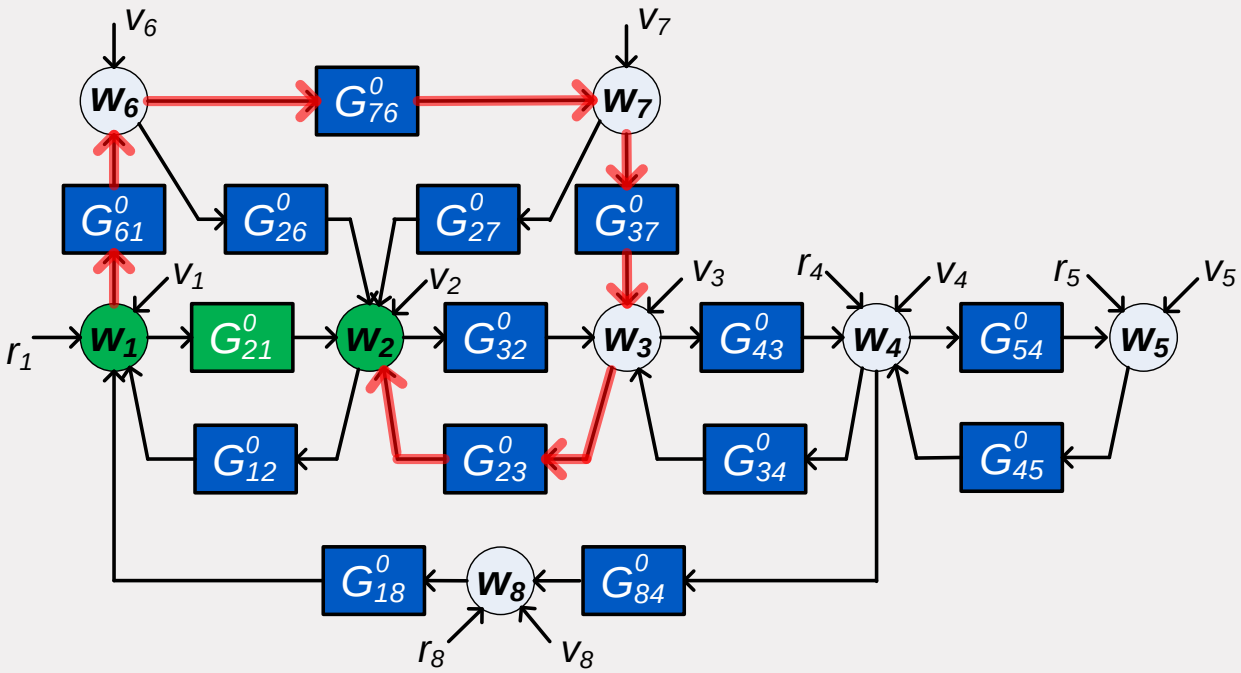
Single module identification

parallel paths, and loops around the output



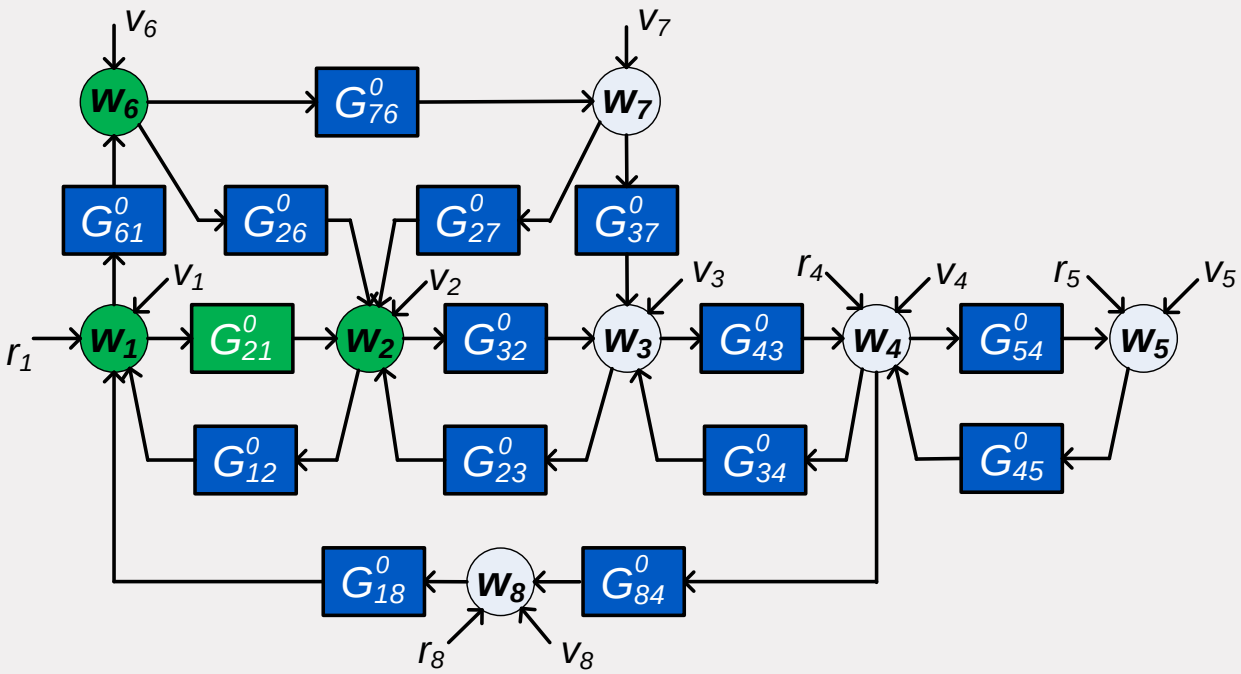
Single module identification

parallel paths, and loops around the output



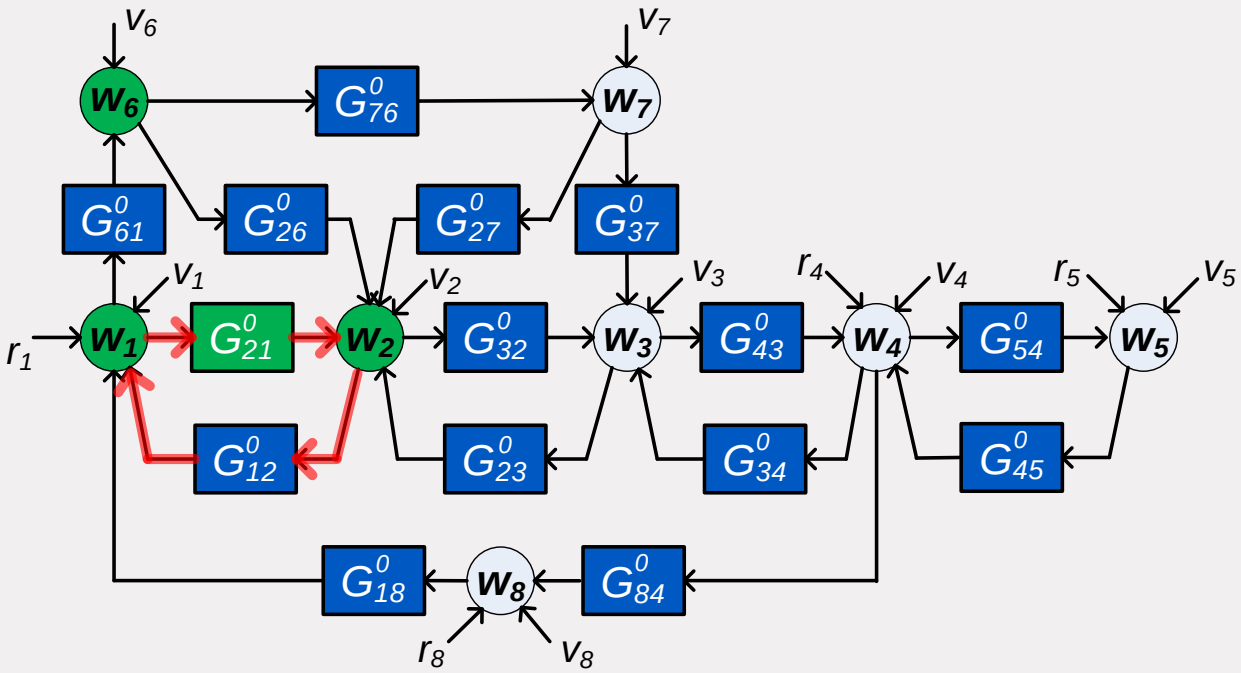
Single module identification

Choose w_6 as an additional input (to be retained)



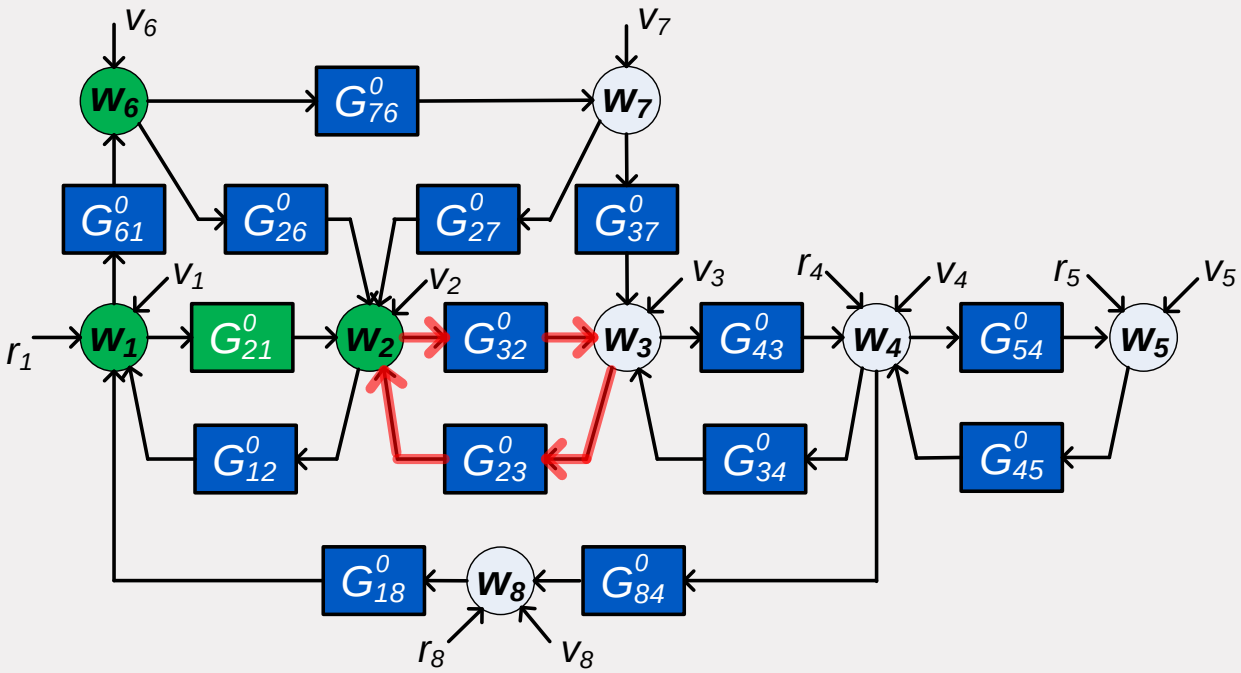
Single module identification

parallel paths, and **loops around the output**



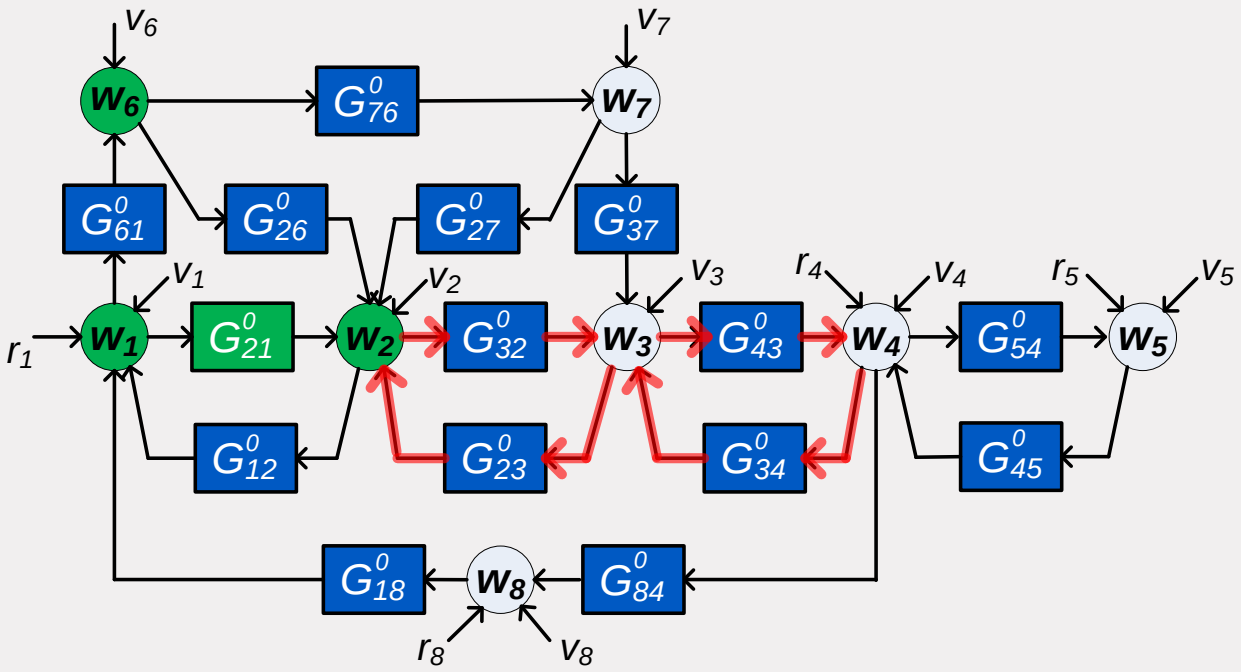
Single module identification

parallel paths, and **loops around the output**



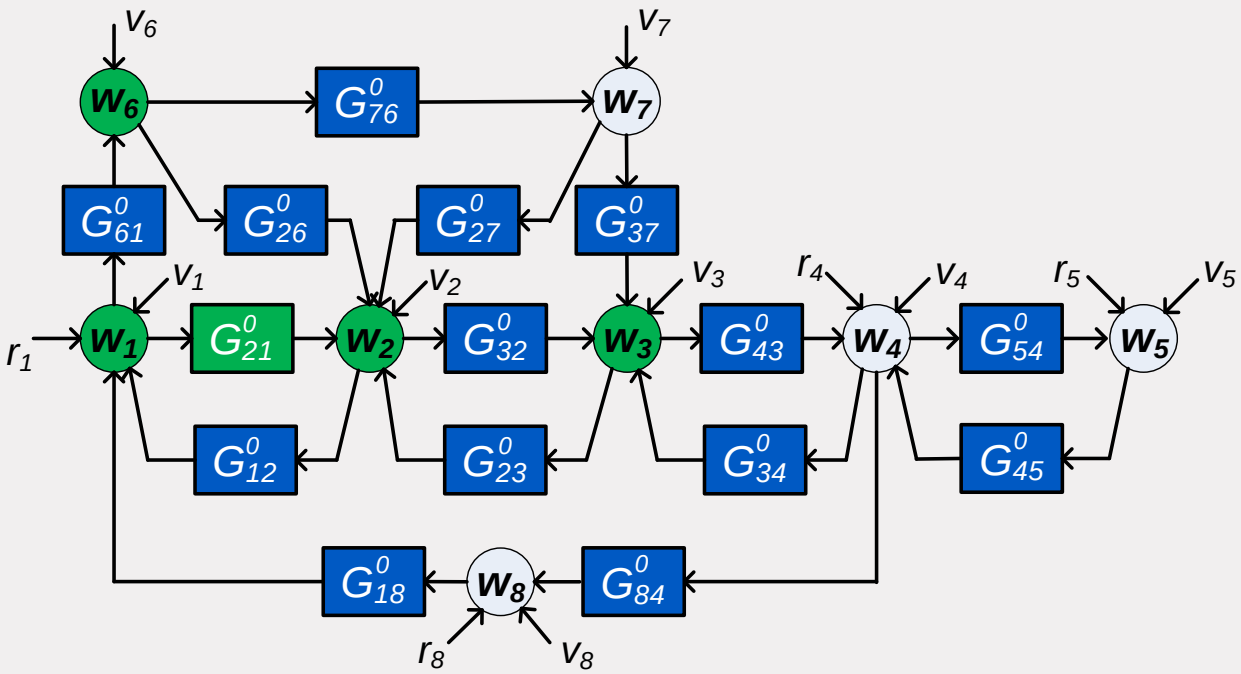
Single module identification

parallel paths, and **loops around the output**



Single module identification

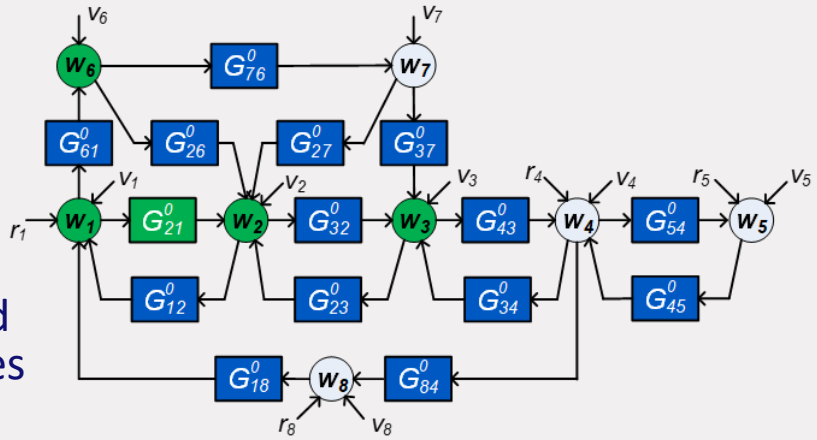
Choose w_3 as an additional node, to be retained



Single module identification

Conclusion:

With a 3-input, 1 output predictor model, the module G_{21}^0 remains invariant.

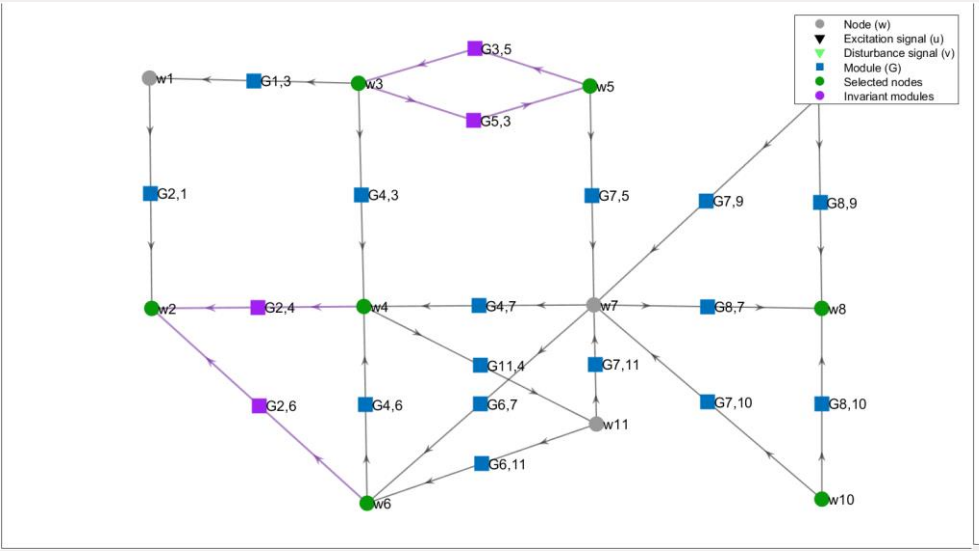


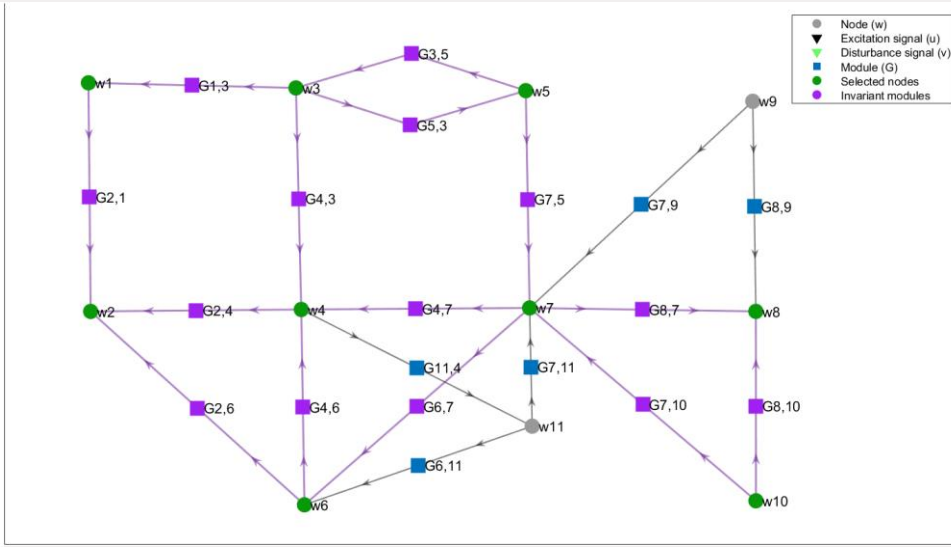
The **indirect method** can directly be applied to this reduced-input situation, and provides consistency

For the **direct method** the properties of the disturbances need to be further investigated

Invariant modules for a given set of measured nodes

For a selected set of measured nodes: algorithm determines with modules remain invariant:

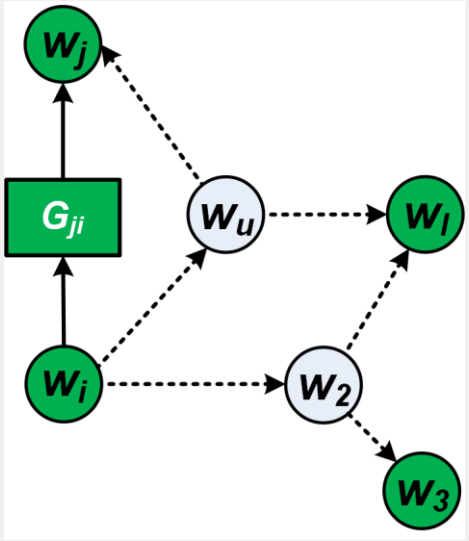




selected nodes invariant modules

Generalization of immersion

Parallel paths and loops around the output can also be blocked by indirect measurements:



- In stead of measuring w_u we measure a descendant of w_u
- Every path from w_i to that descendant needs to be blocked too
- Indirect measurements may lead to non-proper modules in the 'immersed' network

see e.g. Linder and Enqvist[1], Gevers et al.[2] , Weerts et al.[3]

[1] J. Linder and M. Enqvist. *Int. J. Control*, 2017.
[2] A. Bazanella, M. Gevers et al., CDC 2017.
[3] H. Weerts, J. Linder, M. Enqvist & PVdH, Automatica, 2020

Generalization of immersion - Abstraction

The applied reasoning is to exploit the degrees of freedom in the network representation:

$$\begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \\ w_{\mathcal{V}} \\ w_{\tilde{\mathcal{Z}}} \end{bmatrix} = \begin{bmatrix} G_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}} & G_{\tilde{\mathcal{S}}\mathcal{L}} & G_{\tilde{\mathcal{S}}\mathcal{V}} & G_{\tilde{\mathcal{S}}\tilde{\mathcal{Z}}} \\ G_{\mathcal{L}\tilde{\mathcal{S}}} & G_{\mathcal{L}\mathcal{L}} & G_{\mathcal{L}\mathcal{V}} & G_{\mathcal{L}\tilde{\mathcal{Z}}} \\ G_{\mathcal{V}\tilde{\mathcal{S}}} & G_{\mathcal{V}\mathcal{L}} & G_{\mathcal{V}\mathcal{V}} & G_{\mathcal{V}\tilde{\mathcal{Z}}} \\ G_{\tilde{\mathcal{Z}}\tilde{\mathcal{S}}} & G_{\tilde{\mathcal{Z}}\mathcal{L}} & G_{\tilde{\mathcal{Z}}\mathcal{V}} & G_{\tilde{\mathcal{Z}}\tilde{\mathcal{Z}}} \end{bmatrix} \begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \\ w_{\mathcal{V}} \\ w_{\tilde{\mathcal{Z}}} \end{bmatrix} + \begin{bmatrix} u_{\tilde{\mathcal{S}}} \\ u_{\mathcal{L}} \\ u_{\mathcal{V}} \\ u_{\tilde{\mathcal{Z}}} \end{bmatrix} + \begin{bmatrix} v_{\tilde{\mathcal{S}}} \\ v_{\mathcal{L}} \\ v_{\mathcal{V}} \\ v_{\tilde{\mathcal{Z}}} \end{bmatrix},$$

- $i, j \in \tilde{\mathcal{S}}$ as well as other measured nodes
- nodes in $\mathcal{V}$ are not measured but indirectly observed by nodes in $\mathcal{L}$
- Remove $w_{\tilde{\mathcal{Z}}}$ by solving for the 4th equation.
- Remove $w_{\mathcal{V}}$ by solving for the 2nd equation.

$$\begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \end{bmatrix} = \begin{bmatrix} \check{G}_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}} & \check{G}_{\tilde{\mathcal{S}}\mathcal{L}} \\ \check{G}_{\mathcal{L}\tilde{\mathcal{S}}} & \check{G}_{\mathcal{L}\mathcal{L}} \end{bmatrix} \begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \end{bmatrix} + \begin{bmatrix} \check{u}_{\tilde{\mathcal{S}}} \\ \check{u}_{\mathcal{L}} \end{bmatrix} + \begin{bmatrix} \check{v}_{\tilde{\mathcal{S}}} \\ \check{v}_{\mathcal{L}} \end{bmatrix}.$$

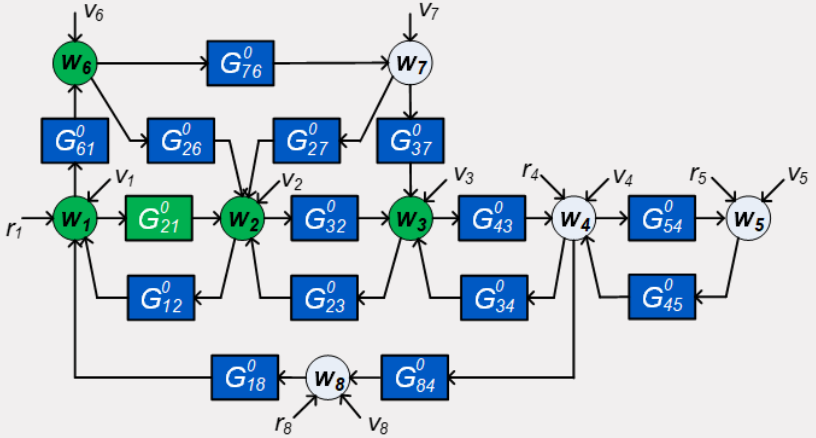
Determine conditions under which $\check{G}_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}} = G^0_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}}$.

Single module identification – indirect method

Observation:

With a 3-input, 1 output model we can consistently identify G_{21}^0

with an indirect method:



with $\mathcal{D} = \{1, 3, 6\}$; $\mathcal{Y} = \{2\}$:

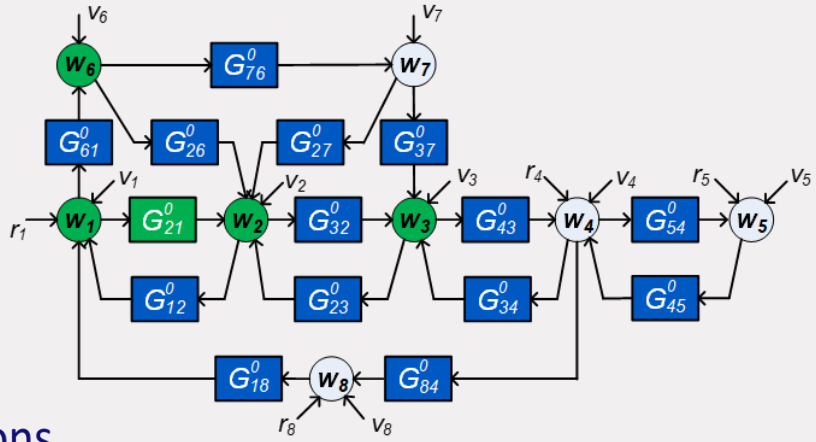
- Let $r_{\mathcal{D}}$ be external excitation signals entering the input node signals $w_{\mathcal{D}}$;
- Then estimate the transfer functions $T_{w_{\mathcal{D}} r_{\mathcal{D}}} : r_{\mathcal{D}} \to w_{\mathcal{D}}$

  $T_{w_{\mathcal{Y}} r_{\mathcal{D}}} : r_{\mathcal{D}} \to w_{\mathcal{Y}}$
- Determine the target module estimate $\hat{G}_{21}$ in: $\hat{G}_{\mathcal{Y}\mathcal{D}} := \hat{T}_{w_{\mathcal{D}} r_{\mathcal{D}}}^{-1} \cdot \hat{T}_{w_{\mathcal{Y}} r_{\mathcal{D}}}$.

Single module identification – indirect method

Indirect method:

$$\hat{G}_{y\mathcal{D}} := \hat{T}^{-1}_{w_{\mathcal{D}} r_{\mathcal{D}}} \cdot \hat{T}_{w_y r_{\mathcal{D}}}$$



- Can be applied irrespective of noise correlations
- Relies on solving 'standard open loop' type of identification problems
- Selection of input signals requires knowledge of the network topology
- Requires an independent r-signal for each input (expensive)
- Does not exploit excitation properties of disturbances

Single module identification

Next step:

Generalize the direct method, to be able to deal with:

- Selections of measured node signals
- Can deal with disturbances with nondiagonal $\Phi_v(\omega)$

so as to exploit the excitation properties of the disturbances.

This is going to be addressed in the local direct method

TU/e